FOURIER ANALYSIS OF EXPERIMENTAL FINITE-AMPLITUDE STANDING WAVES

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THESIS

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by

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ABSTRACT

Finite-amplitude standing waves in air at ambient conditions contained in a rigid-walled cylindrical tube with a large length-to-diameter ratio were experimentally investigated. The pressure waveform at the end of the tube was digitized and Fourier analyzed on an IBM 360 digital computer. Amplitudes and phases were obtained for all harmonics with amplitudes greater than 1% of the fundamental for strength parameters from 0.25 to 1.00 and for frequency parameters from -0.8 to 2.0. The strength and frequency parameters are defined as MbQ and $2\Delta f/\Delta f_{\frac{1}{2}}$ respectively, where M is the Mach number of fundamental, b the nonlinearity parameter, Q the quality factor of the resonator, Δf the frequency away from fundamental resonance, and $\Delta f_{\frac{1}{2}}$ the band width at the half-power points. When these results are compared to the theoretical model of Coppens and Sanders, it is seen that while the theory accurately predicts the magnitude and shape of the harmonic content, it consistently underestimates the frequency at which each harmonic peaks. In addition, the theory fails to predict, except qualitatively, the phase angles.

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I. INTRODUCTION

Coppens and Sanders [1,2] have developed a one-dimensional, nonlinear wave equation with dissipative terms corresponding to those encountered in a rigid-walled closed tube with a large length-to-diameter ratio. An attenuation constant of the form

$$\alpha_n = \alpha_1/\sqrt{n}$$

is assumed, where α_n is the absorption coefficient associated with the frequency $n\omega$ which characterizes the wall losses, R is the radius of the tube and L is the length of the tube. With this additional loss mechanism the wave equation takes the form

$$\sum_{n=1}^{\infty}\left(\frac{\partial^2}{\partial x^2} - \frac{1}{C_o^2}\frac{\partial^2}{\partial t^2} + D_n\right) U_n = b\frac{\partial^2}{\partial x \partial t}\left(\frac{\partial \xi}{\partial x}\right)^2,$$

where

$$b = (\gamma+1)/2 = (C_p/C_v+1)/2,$$

$$D_n = \delta_1\left(\frac{1}{\omega n^{3/2}}\frac{\partial^3}{\partial x^2 \partial t} - \frac{1}{n^{1/2}}\frac{\partial^2}{\partial x^2}\right),$$

$$\delta_1 = \frac{2}{R}\left(\frac{\nu}{2\omega}\right)^{\frac{1}{2}}\left[1 + \frac{\gamma-1}{(Pr)^{\frac{1}{2}}}\right],$$

γ = C_p/C_v the ratio of specific heats,

ν = kinematic viscosity,

Pr = Prandtl number,

ω = angular frequency of the fundamental,

n = harmonic number,

C_o = the thermodynamic speed of sound,

α_1 = absorption coefficient for the lowest frequency

and

U = particle velocity = $\frac{\partial \xi}{\partial t} + \sum_n U_n$.

To arrive at the above equation it was necessary to make the simplifying assumptions that the wall losses were much greater than the bulk losses and that the Mach number

$$M = P_1/\rho_o C_o^2$$

is much less than unity. P_1 is the peak value of the fundamental of the pressure at the rigid end of the tube.

A Fourier series solution of the form

$$\frac{P(x_1 t)}{\rho_o C_o^2} = M \sum_{n=1}^{\infty} R(n) \operatorname{Cos}\left[n\pi(1-\frac{x}{L})\right] \operatorname{Sin}(n\omega t+\phi n)$$

is assumed with the tube excited with a pressure distribution given by

$$P_1(x_1 t) = M\rho_o C_o^2 \operatorname{Cos}\pi(1-x/L)\operatorname{Sin}\omega t.$$

When this solution was substituted directly into the nonlinear wave equation, the following set of algebraic equations resulted:

$$H_n\left[(\operatorname{Cos}\theta_n)S_n - (\operatorname{Sin}\theta_n)C_n\right]$$

$$= \frac{n}{2\ell_s}\left\{\frac{1}{2}\sum_{j=1}^{n-1}(S_{n-j}C_j+C_{n-j}S_j) - \sum_{j=1}^{\infty}(S_{n+j}C_j-S_{n+j}S_j)\right\}$$

and

$$H_n\left[(\mathrm{Cos}\,\theta_n)C_n + (\mathrm{Sin}\,\theta_n)S_n\right]$$

$$= \frac{n}{2\ell_s}\left\{\frac{1}{2}\sum_{j=1}^{n-1}(C_{n-j}C_j - S_{n-j}S_j)\right.$$

$$\left. - \sum_{j=1}^{\infty}(C_{n+j}C_j + S_{n+j}S_j)\right\},$$

where

$$S_n = R_n \mathrm{Sin}\,\phi_n, \quad C_n = R_n \mathrm{Cos}\,\phi_n,$$

$$\ell_s = 2/SP,$$

$$\theta_n = \mathrm{Tan}^{-1}\left[\frac{1 - n^{-\frac{1}{2}} - 2\Delta\omega/\omega_r\delta_1}{n^{-\frac{1}{2}}tn\alpha_2/\alpha_1}\right]$$

$$= \mathrm{Tan}^{-1}[A/B]_1$$

$$H_n = (A^2+B^2)^{\frac{1}{2}},$$

α_2 = infinitesimal amplitude absorption coefficient in free space,

ω_r = angular frequency at resonance,

SP = strength parameter

$$SP = Mbk/2_1 = P_1bk/(2\rho_oC_o^2\alpha_1)$$

k = wave number

and

$$\Delta\omega = \omega - \omega_r.$$

These equations were then programmed for computer solution. For given values of $\Delta\omega$ and SP, the computer calculated values of P_n/P_1 and θ_n. At present, solutions have been found for strength parameters up to 0.75.

Previous investigations in this area have been carried out by Sanders [1,2] and Beech [3]. Both investigators examined the harmonic content of the pressure waveform at the rigid end of the tube near resonance conditions. Sanders employed a wave analyzer to obtain the frequency response for the first four overtones. Good agreement with theory was found but no phase information was obtained. Beech used photographic and graphical techniques to provide numerical analysis of pressure waveforms near resonance. Phase information, however, was obtained only for cases at resonance. Again good agreement with theory was obtained for cases approaching the onset of shock.

The present investigation examines both the harmonic content and phase relationships for strength parameters as high as 0.75. Again numerical methods were employed. Whereas the graphical techniques of Beech were used to generate 64 data equally spaced throughout the pressure wave form, the technique in this investigation employed analog-to-digital conversion by computer to provide better than 1000 data to represent the pressure waveform. With the pressure waveform digitized to such an extent, it was possible to obtain amplitude and phase information for all harmonics with amplitudes greater than one percent of the fundamental.

II. EXPERIMENTAL CONSIDERATIONS

A. APPARATUS

A block diagram of the experimental system used in this investigation is shown in Figure 1. Finite-amplitude standing waves were generated in air at ambient conditions contained within a tube six-feet long with an inner diameter of 2.250 in. and a wall thickness of 1.125 in. The air was excited at one end by a piston which was driven by an M-B Electronics Model EA 1500 exciter capable of a maximum no-load acceleration of 124g. The EA 1500 was driven by two M-B electronics Model 2120MB power amplifiers operating in parallel. The maximum attainable acceleration with the weight of the piston as load was about 50g. The frequency and power level were controlled by a General Radio Type 1161-AGC coherent decade frequency synthesizer. With this synthesizer it was possible to select frequency to better than 0.001 cycles/sec. (Monitoring the frequency with a Hewlett-Packard Model 521C frequency counter showed the frequency stability to be better than one part in 10 million.) The acceleration of the piston was determined by measuring the output an Endeveco Model 2215 accelerometer implanted within the piston. When the piston was withdrawn from the tube and driven in free air at the approximate experimental frequency and acceleration, the harmonic distortion was measured to be less than 0.3 percent. The other end of the

tube was capped with a thick plate which was securely bolted to the tube. The sound pressure level at the rigid end of the tube was detected with a 1/4-in. diameter Bruel and Kjaer Type 4136 condenser microphone mounted in the cap so that its diaphragm was flush with the end of the tube. The microphone response was flat to within ± 0.5 dB from 80 Hz to 20 kHz. The outputs of the microphone and accelerometer were monitored with Hewlett-Packard 400D vacuum-tube voltmeters. The microphone output also went to a Hewlett-Packard Model 302A wave analyzer so that the first two overtones could be monitored. The microphone output was recorded by a PI 6200 General Purpose Portable Instrumentation Tape Recorder. Recordings were made in the FM mode at a tape speed of 37.5 in/s so that the frequency response of the tape recorder was ± 1 dB from DC to 10 kHz. Although the signal was recorded at 37.5 in/s it was necessary when digitizing the signal to play back at a speed of 3.75 in/s. To insure that this change in tape speeds did not alter the recorded frequencies, a test signal was recorded at the higher speed and then played back at the slow speed. The two signals were observed on a dual trace oscilliscope where it was observed that the frequency of the test signal was 10.0 times that of the signal played back at 3.75 in. per second with no discernable drift between the two signals.

B. MICROPHONE CALIBRATION

Investigation of the standing waves at various strength parameters requires an absolute calibration of the microphone

system. The microphone system is defined here as the microphone, preamplifier and the equipment loaded onto the output of the preamplifier. The system sensitivity is defined as

$$S_m = V_m/P$$

where V_m is the peak voltage output of the preamplifier and P is the peak pressure at the microphone. Two methods were employed to determine S_m. The first and simplest used a Bruel and Kjaer Model 4220 pistonphone which produces a known sound pressure level at 250 Hz. The sensitivity was found to be $S_m = (-77.1 \pm 0.2)$dB re 1 volt/μbar, where the uncertainty is that inherent in the pistonphone specifications. The second method makes use of theoretical predictions by Coppens for small strength parameters as verified by Beech [3]. Coppens shows that at resonance the ratio of the peak pressure of the second harmonic to the square of the peak pressure of the fundamental is given by

$$P_2/P_1^2 = (\sqrt{2}\ \omega\beta)/(8\rho_o C_o^3\alpha).$$

When the definition of sensitivity is substituted into the above equation the result is

$$S_m = (\beta\omega V_1^2)/(4\sqrt{2}\ \rho_o C_o^3\alpha V_2).$$

S_m was then found by exciting the tube at resonance and measuring the fundamental and second-harmonic voltages. By this method $S_m = (-77.2 \pm 0.3)$dB re 1 volt/μbar where the uncertainty is due largely to the error in determining the attenuation constant.

C. MICROPHONE OUTPUT AND STRENGTH PARAMETER

In this investigation, where the pressure waveform was to be analyzed at various strength parameters, it was necessary to find a relationship between the strength parameter and the observable quantities. Using the equations

$$S_m = SPL + 21 \log\left(\frac{\nu}{.775}\right) + 72.0,$$

$$M = \frac{P_1}{\rho_o C_o^2} = \frac{P_1}{P_o} \cdot \frac{P_o}{\rho_o C_o^2} = SPL - 197.0,$$

and

$$M = \frac{SP(2\alpha/K)}{b},$$

one can show that the rms value of the fundamental of the microphone output (v) is related to strength parameter and the attenuation constant by

$$v = 136.0\, \alpha \cdot SP.$$

The constant 136.0 is relatively insensitive to small temperature variations about room temperature.

D. ATTENUATION CONSTANT

The attenuation constant for infinitesimal amplitudes has been shown by Beech to be

$$\alpha_1 = \left(\frac{\pi}{C_o}\right) \Delta f,$$

where Δf is the frequency difference between the half power points on the resonance curve (taken for constant acceleration amplitude of the piston).

E. FREQUENCY PARAMETER

The frequency parameter is defined by

$$FP = 2\Delta\omega/\omega_r\delta_1,$$

where

$$\Delta\omega = \omega - \omega_r$$

and

$$\delta_1 = 2\alpha_1 C_o/\omega_r.$$

This frequency parameter is related to the experimental frequency increment (Δf_{ex}) by

$$\Delta f_{ex} = \Delta f \cdot FP/2,$$

where Δf is the frequency difference between the half power points of the infinitesimal resonance curve as used in calculating α_1 in this investigation Δf was found to be $1.15 \pm 0.01H_3$ so that

$$\Delta f_{ex} = 0.58 \ FP.$$

III. COMPUTER ANALYSIS

A. SAMPLE RATE

Many computer programs exist which, when supplied with data points defining a periodic waveform, will give the Fourier coefficients. The first consideration, generally, is the number of data required per cycle. Theoretically this number should be at least twice the number of harmonics desired. However, the question of accuracy of the computer predictions of harmonic amplitudes arises immediately. As might be expected, the accuracy of the harmonics determined tails off faster as fewer samples per cycle are used. This then leads to the problem of first determining how many significant harmonics are desired. For finite-amplitude standing waves of large strength parameter the waveform qualitatively approaches a function whose harmonics content falls off inversely with harmonic number. If such a waveform is passed through amplifiers and storage devices before being analyzed, harmonics below a given amplitude will be lost in the inherent noise. The device in this investigation which was most seriously restrictive in this respect was the PI 6200 FM tape recorder. It was found that the dynamic range of this instrument was about 42 db which restricted the investigation to harmonics with amplitudes greater than about one percent of the fundamental. For a periodic ramp function this restriction occurs at about the 30th harmonic.

Determination of the number of samples per cycle to be used was based on the arbitrary criterion that the error in the 30th harmonic be less than 0.1 percent for an ideal ramp function. Applying several computer algorithms for determining Fourier coefficients to an ideal ramp function one finds that 1000 samples per cycle are required for the above accuracy.

B. THE DIGITAL-COMPUTER PROGRAM

The primary function of this program was to generate normalized Fourier coefficients for a periodic function given digitized data on a seven-track tape. Secondary to this, the program changed the form of the coefficients to $A(n) \mathrm{Sin}(N\omega t + \phi(n)$ and computed $A(n)/A(1)$ for each harmonic. Optional functions provided by the program were the averaging of several periods of data to reduce the affects of random noise. An optional plot of the input data was available along with points reconstructed from the calculated Fourier coefficients. This plot provided a visual check on the input data and the computed coefficients.

The digitized data provided by the SDS 9300 were written on seven-track tape. However, the binary representation of numbers on the seven-track tape differs from those on the standard nine-track tape used by the IBM 360 system. To accomodate the seven-track tape, a conversion program (subroutine FORM) was used to convert the data. The converted data were not floating point but integers and required further conversion with the use of a scaling factor.

Prime consideration was given to the types of algorithms commonly used in generating Fourier coefficients. Two of the more widely used methods are the Cooley-Tukey [5] Fast Fourier transform and the recursive technique of Goertzel [4]. Both methods were tested for accuracy in predicting the coefficients of known waveforms. The two methods were found to be equal. One advantage of the Cooey-Tukey method is its minimal use of computer time. Its computation time is about an order of magnitude less than that required by the Goertzel method. However, when this time saving is compared with the overall required computer time for analysis the advantage diminishes. The final consideration and, in the end the most important, was the flexibility in using data. The Cooley-Tukey method required an integer power-of-two number of points. To be able to satisfy this condition along with the requirement for 1000 or more samples per cycle complicated the digitizing process. The Goertzel method required only an odd number of samples and for this reason was used in the computer program.

C. THE ANALOG TO DIGITAL CONVERSION (A-D)

The conversion of the analog data was accomplished with the use of a hybrid computer consisting of the SDS 9300 high-speed digital computer and the Ci 5000 general-purpose analog computer. Use of the system for A-D conversion requires a FORTRAN program for the SDS 9300 and properly constructed analog and logical patch boards for the Ci 5000. In addition, several peripheral equipments are required.

These are the card reader, magnetic tape transport, line printer and, if desired, a pen recorder.

Aside from the question of sampling, which was determined earlier, the important question in the A-D conversion is accuracy. The clock in the computer is accurate to one part in 100 million, well within any accuracy requirements. The A-D converter is designed to accept signals between $\pm$ 100 volts and divide this interval into 16,384 parts providing a resolution of 12.2 mV. For a properly amplified analog signal, the dynamic range of the A-D converter is much greater than the PI 6200 tape recorder.

The analog patch board (Figure 3), as used for A-d conversion, served to amplify the analog signal prior to conversion and distributed the signal to the converter and oscilliscope. Additionally, a patch was provided linking the Digital-to-Analog converter with a pen recorder. This additional function is explained in detail in the procedures section.

The logic patch board (Figure 4), had three functions. The first provided a sampling frequency (10,000 Hz) to the A-D converter. This frequency represents the approximate maximum sampling rate allowed. Since the approximate resonant frequency of the tube was 94 Hz it was necessary to play back signals from the PI 6200 at one tenth the recorded speeds to acquire the desired number of samples per cycle. The second function provided a triggering voltage through an external switch (DS-1) which was used to start the data

conversion process. Associated with this function is an externally controlled delay (DF00) which insured that the triggering voltage remained on long enough for one set of data (one record) to be written on the seven-track tape. The last function provided another triggering voltage through an external switch (DS-2). When the last record was to be written on tape switch DS-2 was turned on while switch DS-2 was triggered.

D. SYSTEM TEST

Prior to analyzing experimental data, it was necessary to test the entire system process of A-D conversion and computer analysis. The results of this test for a ramp function is shown in Figure 5.

IV. PROCEDURES

A. PRERUN PROCEDURES

Beech pointed out that the resonant frequency of the tube changed by 0.16 Hz for each degree centigrade of temperature change. To minimize this affect, all data gathering runs were made during the evening hours when the room temperature remained relatively constant for several hours. Typical resonant frequency variations during these hours were of the order of 0.01 Hz per hour. Several hours prior to taking data the equipment was turned on and allowed to stabilize. During this warmup period, the PI 6200 tape recorder was calibrated to insure that the FM system was properly aligned and to insure that the signals being recorded were not overdriving the tape recorder preamplifier and thereby introducing unwanted signal distortion. Next, the alignment of the piston in the free end of the tube was checked by observing the output of the accelerometer on an oscilloscope and measuring, with a wave analyzer, the second- and third-harmonic content. If necessary, the harmonic content could be reduced by carefully adjusting the alignment of the piston so that the distortion from any harmonic was less than 2 percent. To minimize leakage of sound from the tube, the piston was fitted with an "O" ring and lubricated. The harmonic distortion introduced by the piston was sensitive to the type of lubricant

used. It was found that a light turbine oil minimized the distortion while providing an adequate seal.

Just prior to taking each set of data, it was necessary to determine the resonant frequency and the attenuation constant. For this measurement the piston was excited at a constant value of acceleration low enough so that the strength parameter at resonance was less than 0.03. From the frequency difference between the half power points, the attenuation constant was determined. The resonance frequency was taken to lie half way between the half power points.

B. DATA TAKING PROCEDURE

Corresponding to the desired strength parameter was a given value of the pressure fundamental as shown earlier. With the piston excitation set to the proper level to keep the pressure fundamental constant, data were recorded at 0.1 Hz intervals for twenty frequencies from 0.6 Hz below resonance to 1.3 Hz above. At each frequency, the signals from the microphone and accelerometer were recorded on the PI 6200 at high speed (37.5 in/s) for about 10 seconds (30 ft.), and the second-harmonic amplitude of the pressure was noted to be later compared with the computer analysis. Since the taking of an entire set of data at one strength parameter required from 30 minutes to an hour, at the end of the run it was necessary to check the resonant frequency and attenuation constant to insure that the variation in resonant frequency was substantially less than one frequency interval (0.1 Hz) and that the change in attenuation constant was less than one percent.

The theoretical predictions (8) are compared with the experimental results in Figures 6 through 13 for each value of strength parameter, plots are shown of the harmonic amplitude vs. frequency parameter and of the phase difference between consecutive harmonics vs. inverse harmonic number. Figure 6 shows the actual data obtained for three different experimental runs. The degree of scatter is a demonstration of the excellent reproducibility of the results. Both experimental and theoretical results are limited to harmonics with amplitude greater than 1% of the fundamental.

In comparing the experimental values of the harmonic amplitudes with the theoretical predictions for a strength parameter of 0.25 it is seen that the agreement is fairly good but that there does seem to be a slight systemat c difference. The corresponding curves for higher strength parameters display this difference to a more marked degree. It is seen that the experimental results peak at about the predicted values of $P_{(n)}/P_{(1)}$ and that the shape of the curve for each harmonic is about as predicted. However, the experimental results peak at a value of frequency parameter significantly greater than predicted and this difference increases with harmonic number. It is interesting to note that at the frequency of maximum distortion of the pressure waveform (at approximately FP = 0.5) theory accurately predicts the harmonic amplitudes.

The comparison between the experimental and theoretical phase angles displays an even greater discrepancy. While

values of $\phi_n - \phi_{n-1}$ do seem to be linearly related to $1/n$, the values for the second harmonic are greatly different and the slopes of the experimental curves are significantly greater than predicted.

In conclusion, it appears that while the present theoretical model is able to predict the magnitude and shape of the harmonic content of the distorted waveform, it consistently underestimates the frequency at which each harmonic peaks. This underestimation increases with harmonic number and strength parameter. In addition, it fails to predict, except qualitatively, the phase relations in the distorted wave.

APPENDIX A

THE A-D PROGRAM PARAMETERS AND OPTIONS

The purpose of this program was to convert up to four channels of analog data into a series of digital points on seven-track magnetic tape. The program had several input parameters which had to be provided to the computer via the teleprinter after the FORTRAN program had been read into the system.

NSAMP (input 1.) was an integer which corresponded to the number of data or samples to be taken from each channel. For this investigation a sampling rate of 10 kHz was used and NSAMP was chosen to span one cycle of the input waveform.

NCHAN (input 2.) was an integer which specified the number of channels of analog data being converted.

NREC (input 2.) specified how many records were to be written on tape each time sampling was initiated. A record was defined as the data set corresponding to the product of NCHAN and NSAMP. Because of the high sampling rate used it was necessary to set this value to 1.

ITAPE (input 4.) designated which magnetic-tape transport held the seven-track tape. The integer used here had to correspond that selected on the "unit select" switch on the particular tape unit used.

NDEL (input 5.) determined how rapidly the data on the seven-track tape was read back when using program option 7.

An example of the input parameters as typed on the teleprinter might be

NREC = 1, NSAMP = 1000, NCHAN = 3, ITAPE = 1,

NDEL = 20* (carriage return).

The control symbol * indicated to the computer that the input parameters had been set. Care was taken when choosing NSAMP and NCHAN to insure that their product did not exceed the first dimension of IBUF found in the first FORTRAN program statement.

When the parameter list had been given the teleprinter responded with

"OPTION = (I1)".

The computer was then given a one-digit number corresponding to the list of available options.

RESTART (option 1.) was used whenever any one or all of the input parameters was to be changed.

GO (option 2.) was used when analog to digital conversion was to be done. Upon receiving this option the computer awaited the initiating trigger from switch DS-1 to start the actual conversion process. Each trigger from DS-1 caused one record of data to be written on tape. When a sufficient number of records were written, DS-1 was triggered with DS-2 in the "up" position. This caused the last record to be written followed by a teleprinter response

"OPTION = (I1)".

END OF FILE (option 3.) was used to put a mark on the tape to separate or isolate groups of records. Groups of records separated by END OF FILE marks are called files.

SKIPFILES (option 5.) were used when files already on tape were to be skipped over. When used, a four-digit number was given the computer to specify the number of files to be skipped.

LIST ONE RECORD (option 6.) was used in reading data from the seven-track tape. Use of this option caused data written on tape to be written on the page printer after having specified the number of data to print.

PLOT ONE RECORD (option 7.), when called, caused one record to be read from the tape, converted to an analog signal, and fed to terminals T420 through T423 on the analog board (corresponding to channels 1 through 4). The signal could then be monitored on various recording devices. Because of the short duration of the signal it was best displayed on the available pen recorder whose input terminals are found on the analog board (Figure 3). The time base of the signal returned from the tape could be lengthened or shortened by varying the value of the input parameter NDEL.

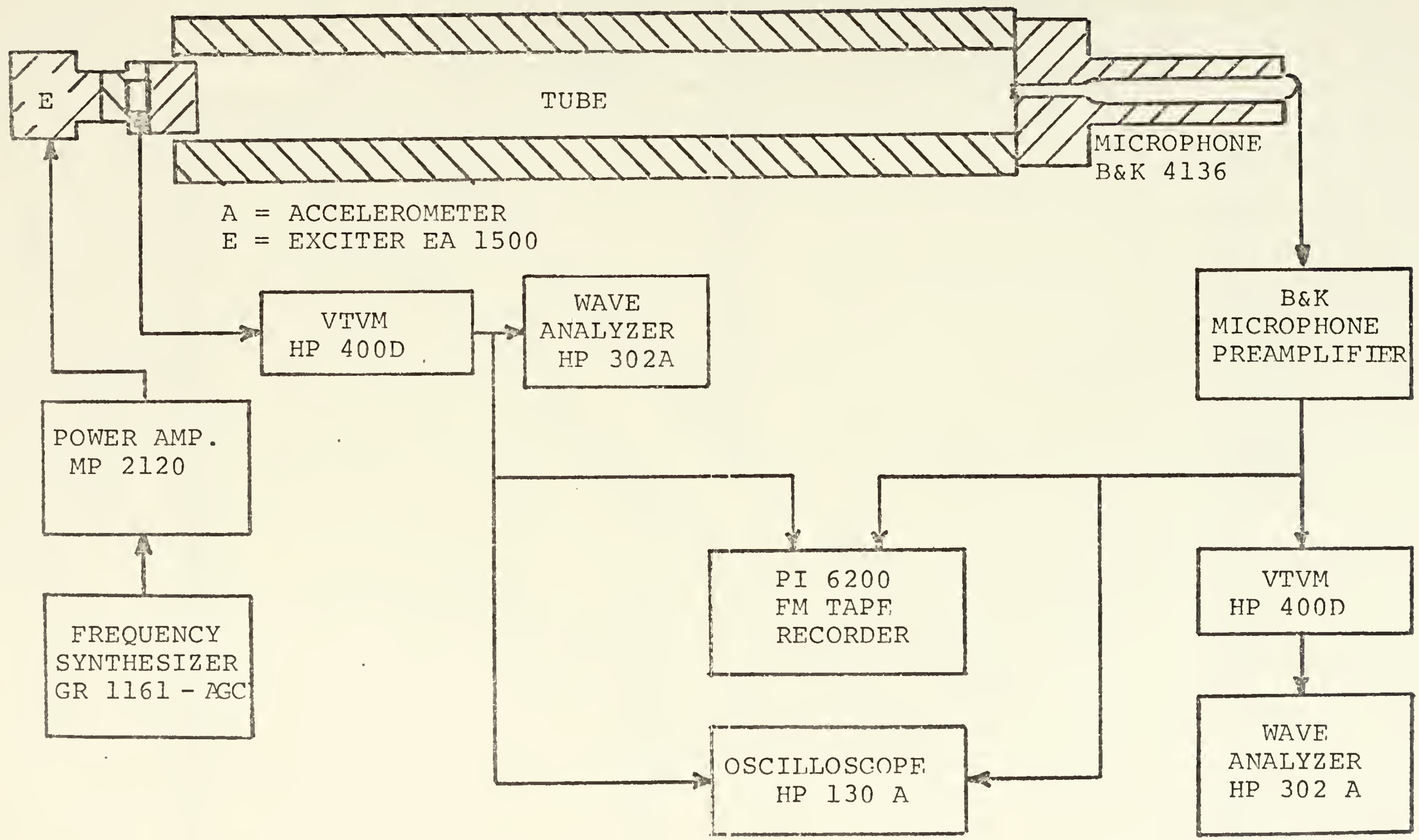


Figure 1. Apparatus.

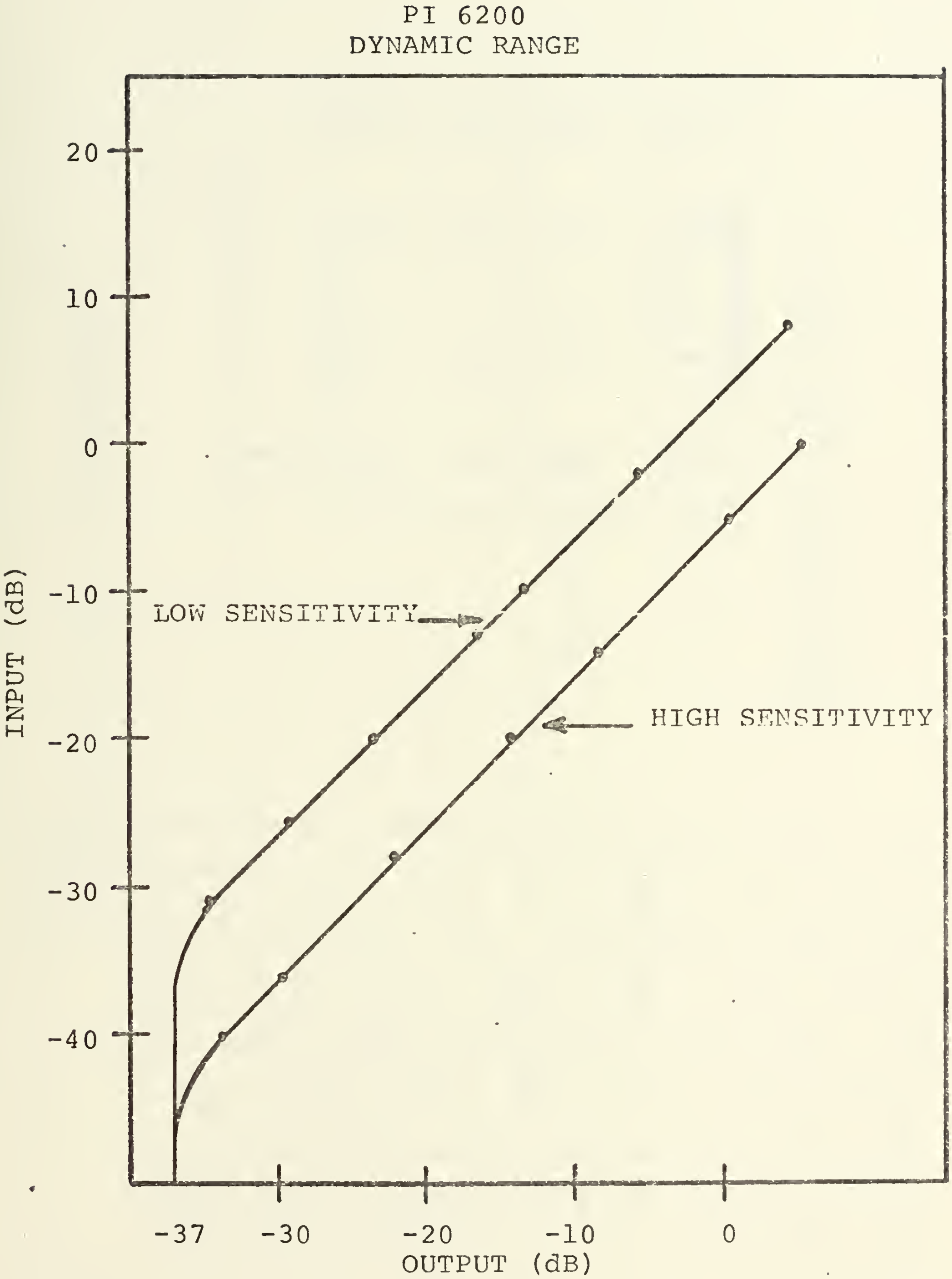


Figure 2. PI 6200 Dynamic Range.

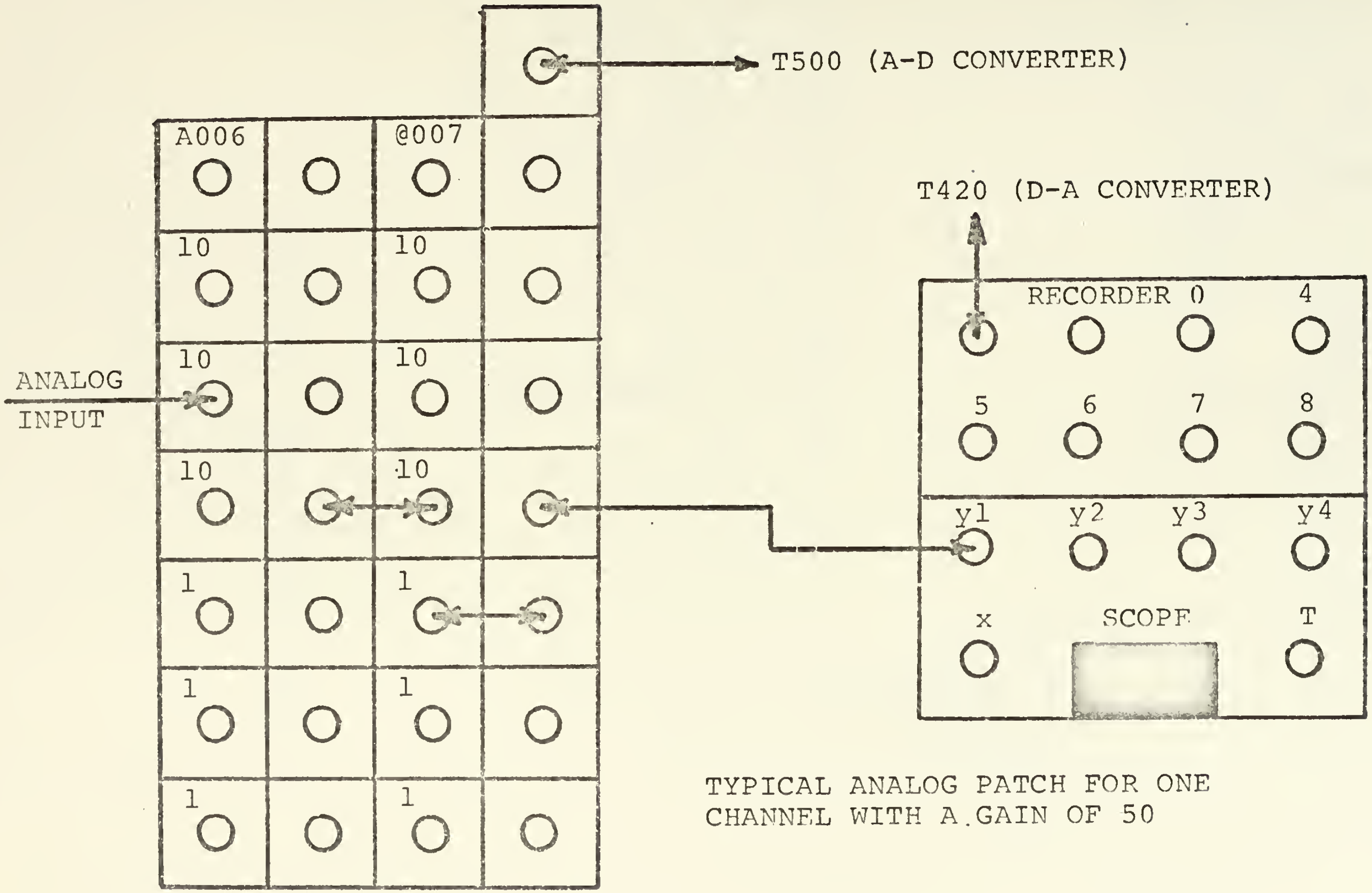


Figure 3. Analog Patch Board.

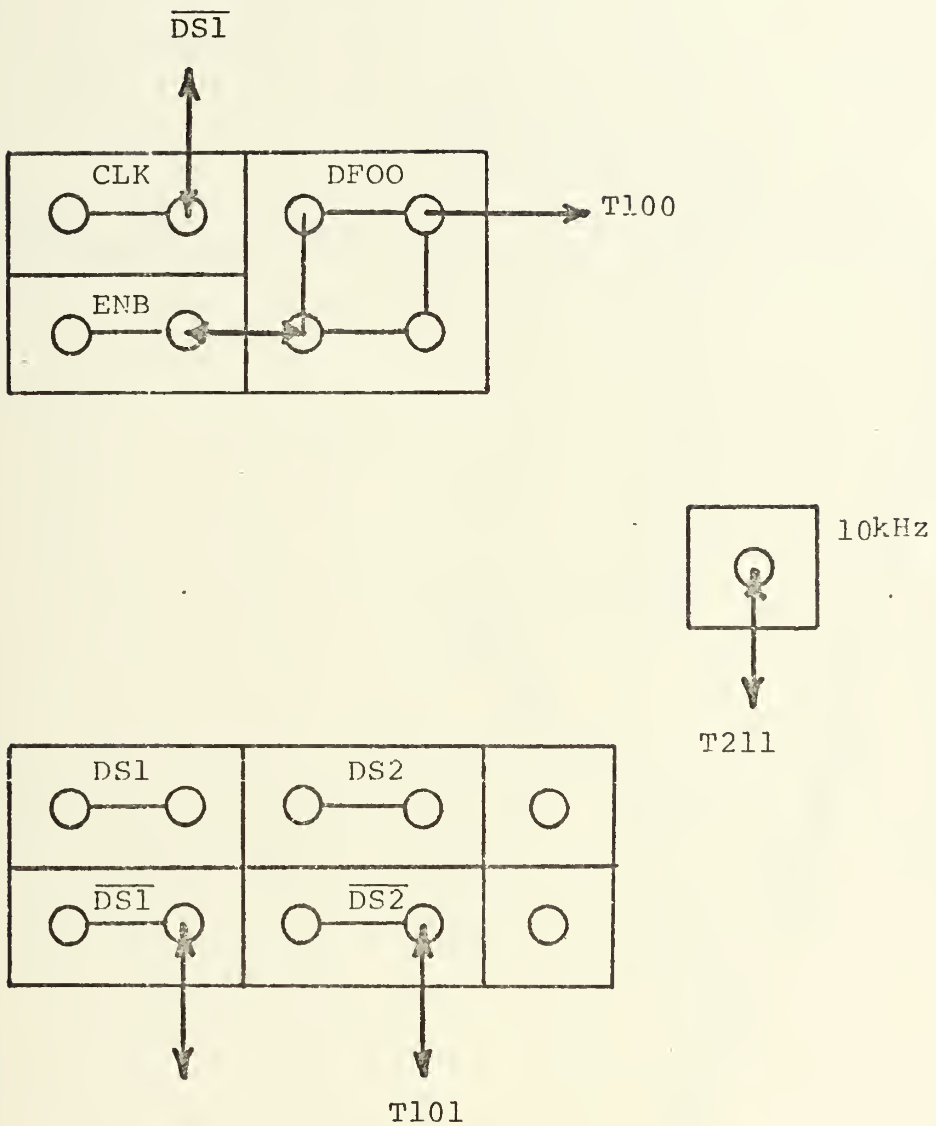


Figure 4. Logic Patch Board.

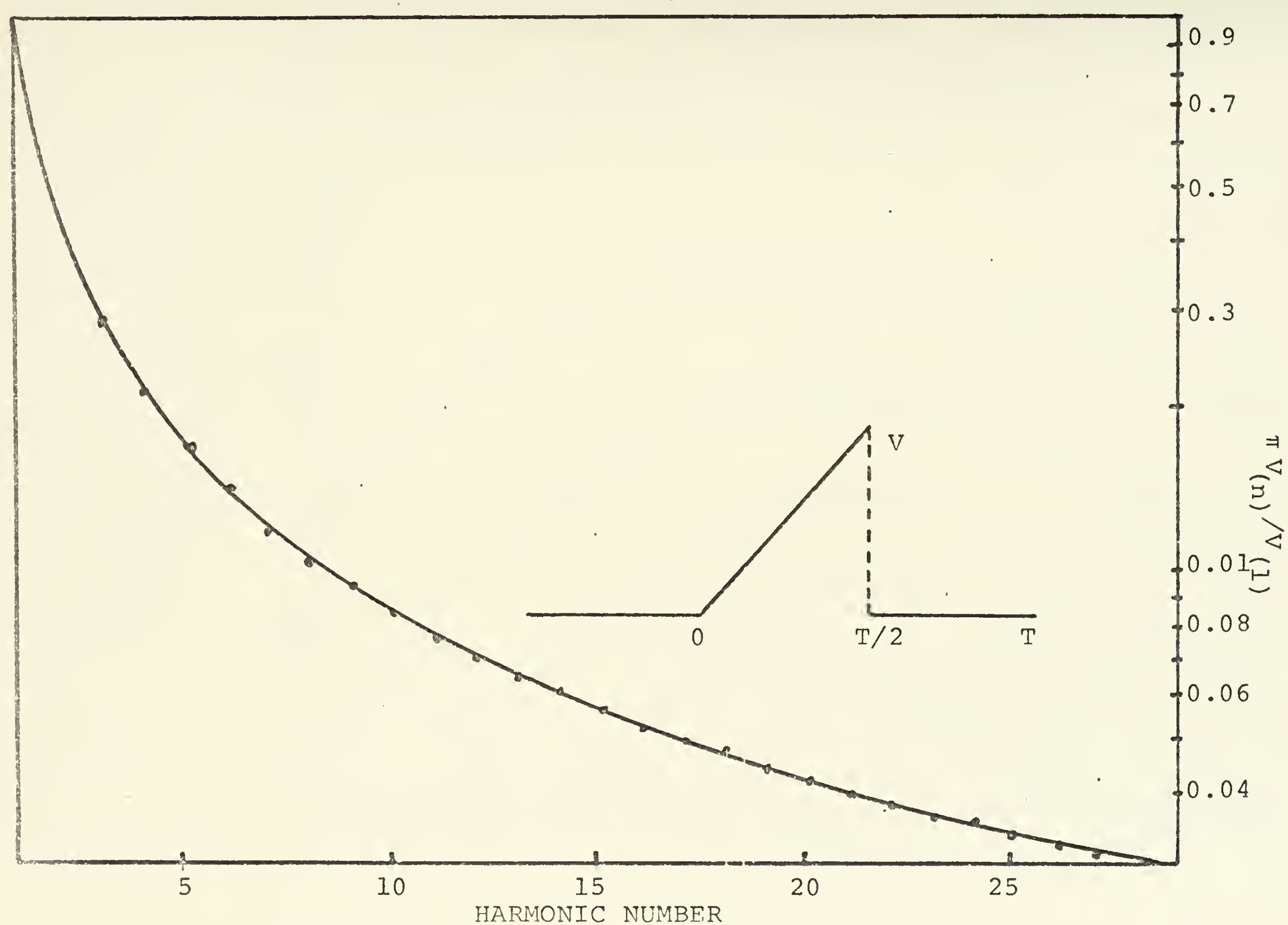


Figure 5. Test Signal Analysis.

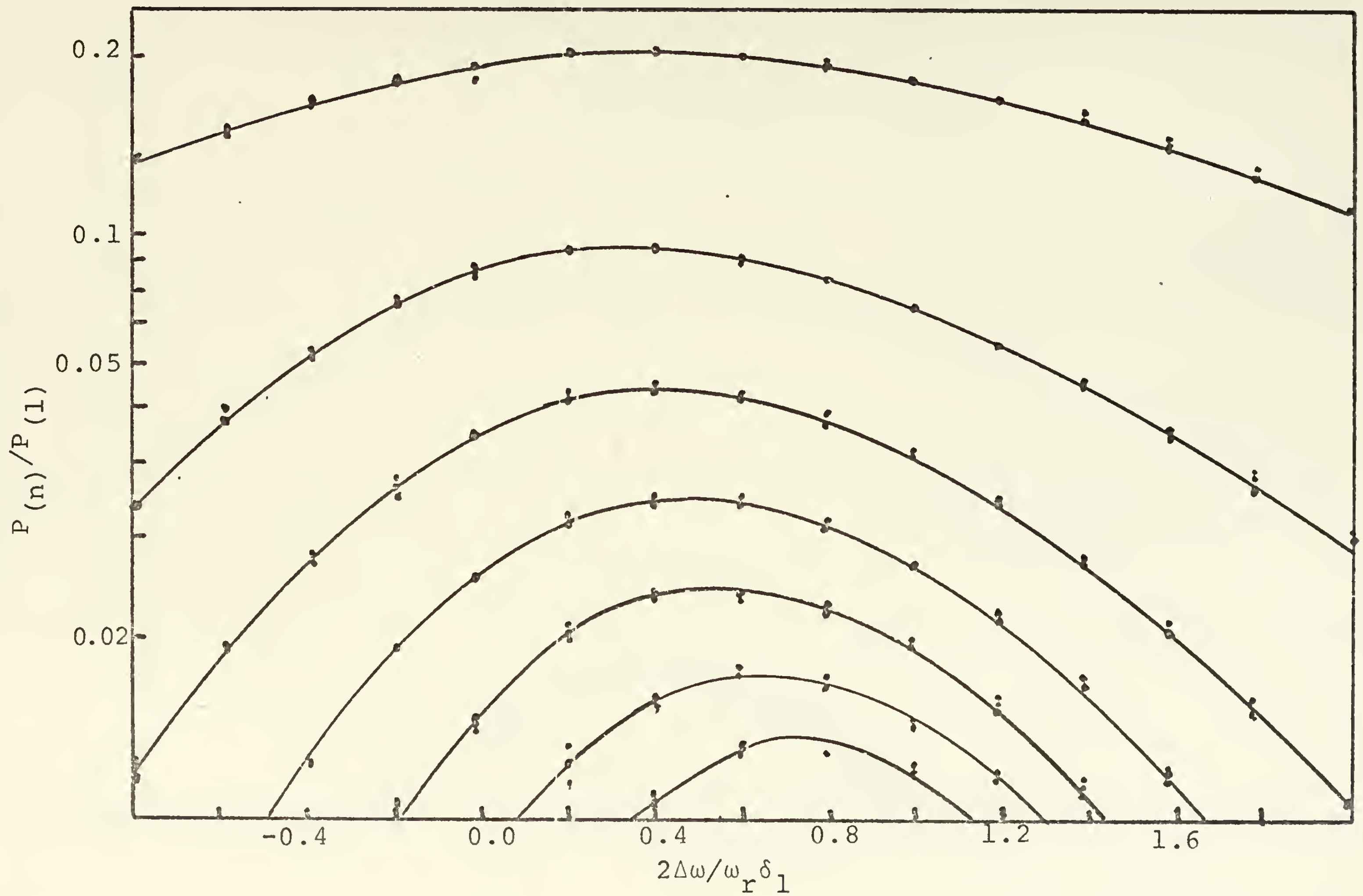


Figure 6. Point Scatter.

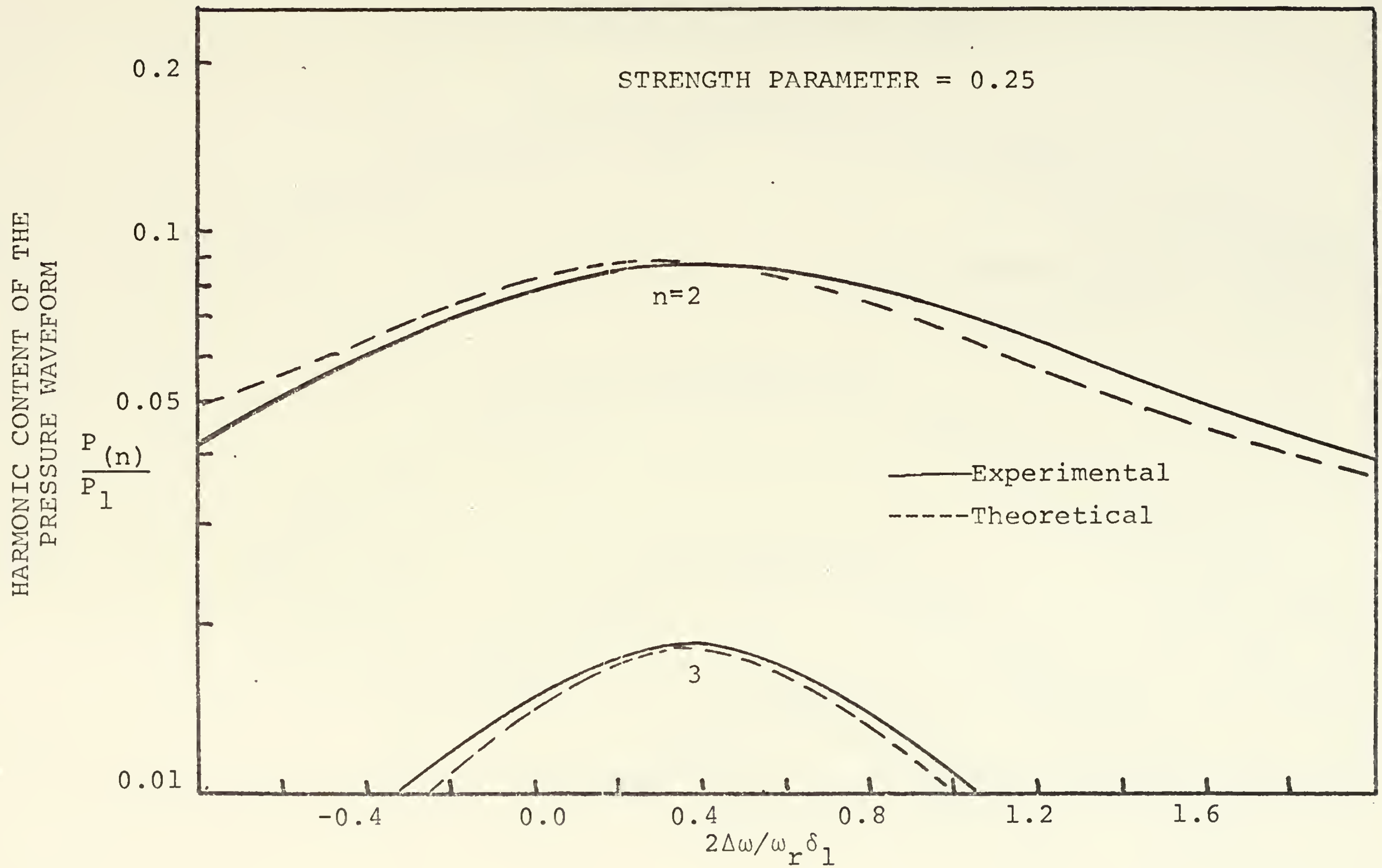


Figure 7. Harmonic Amplitudes SP = 0.25.

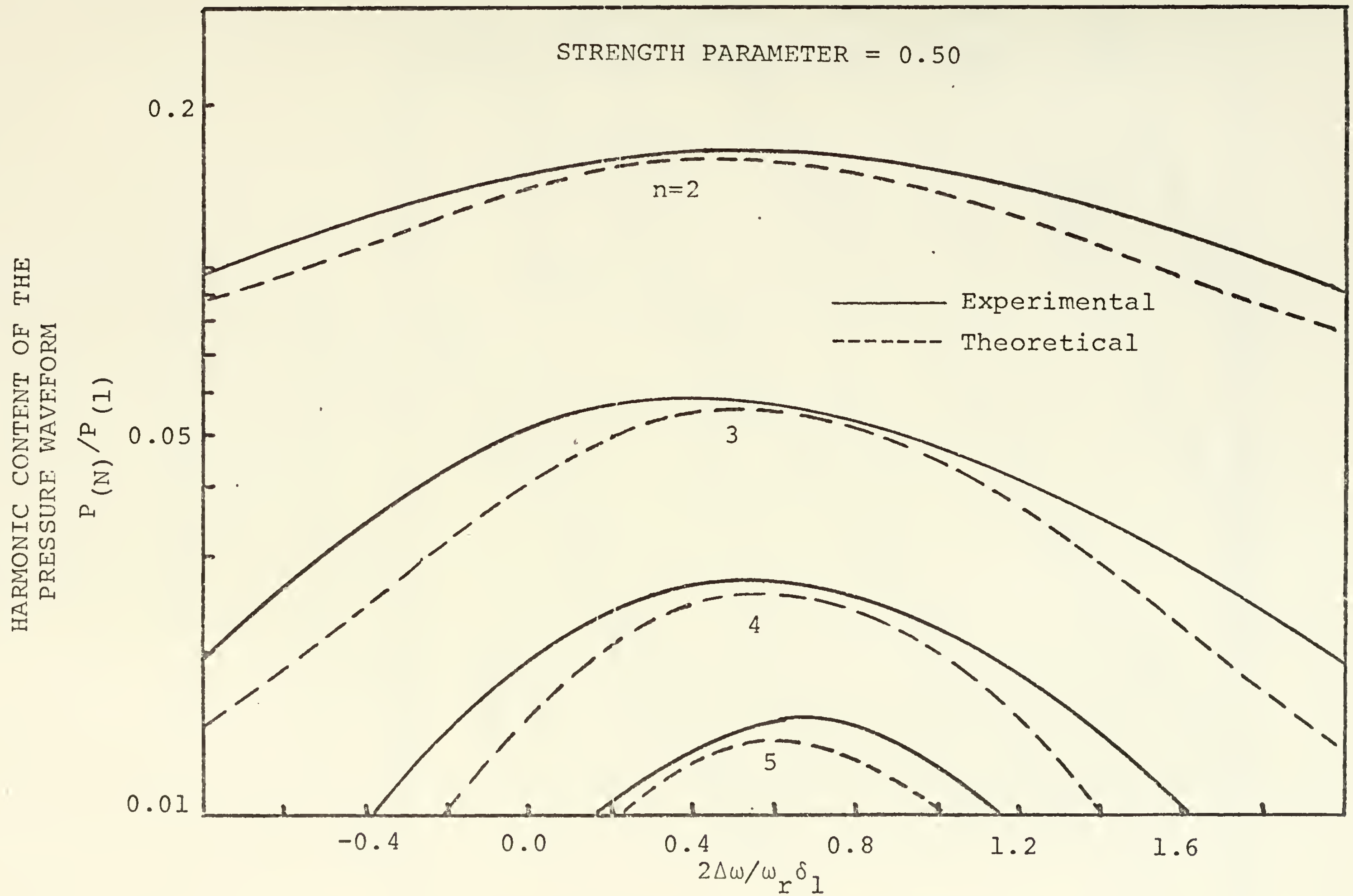


Figure 8. Harmonic Amplitudes SP = 0.50.

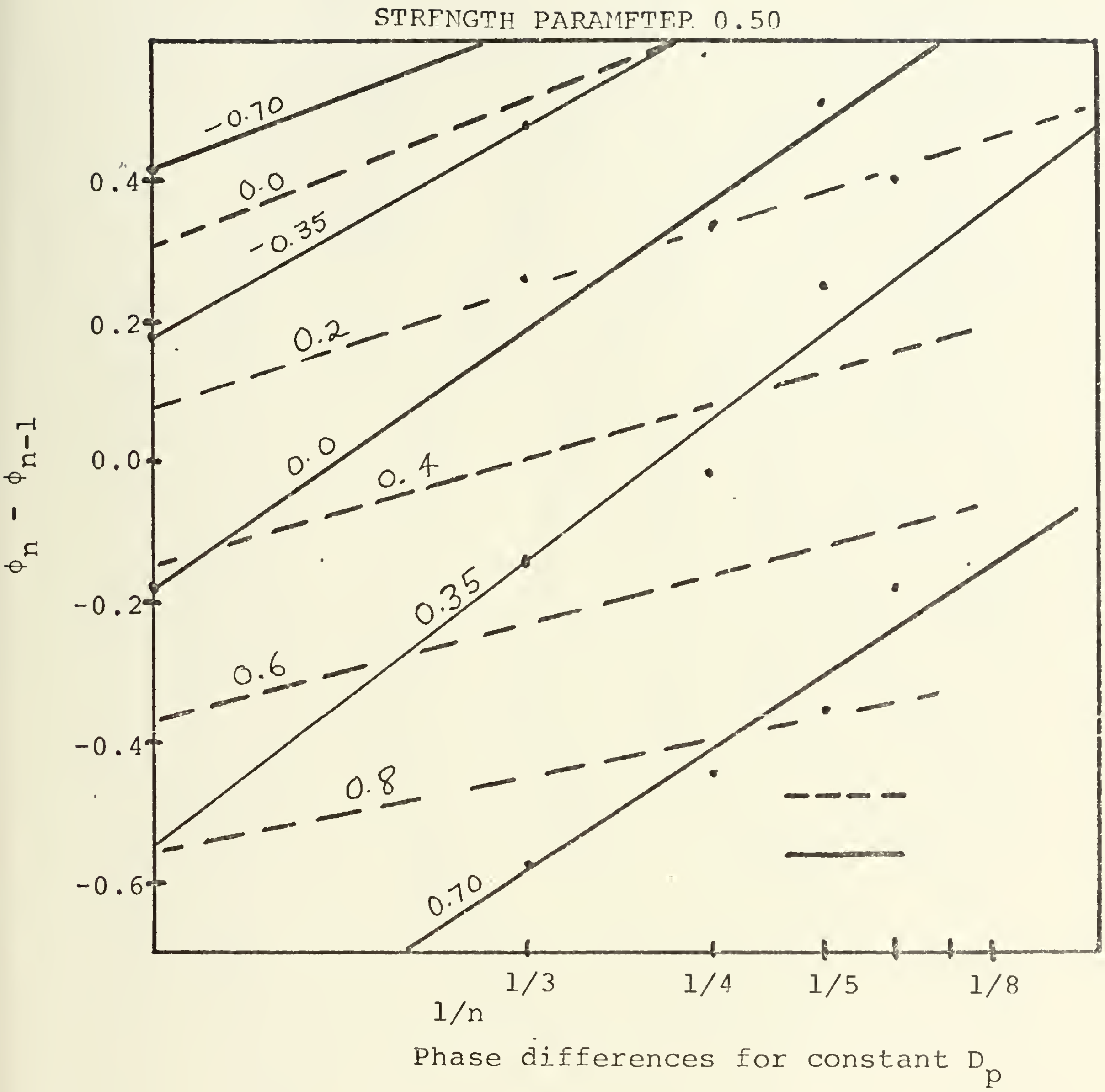


Figure 9. Phase Plot SP = 0.50.

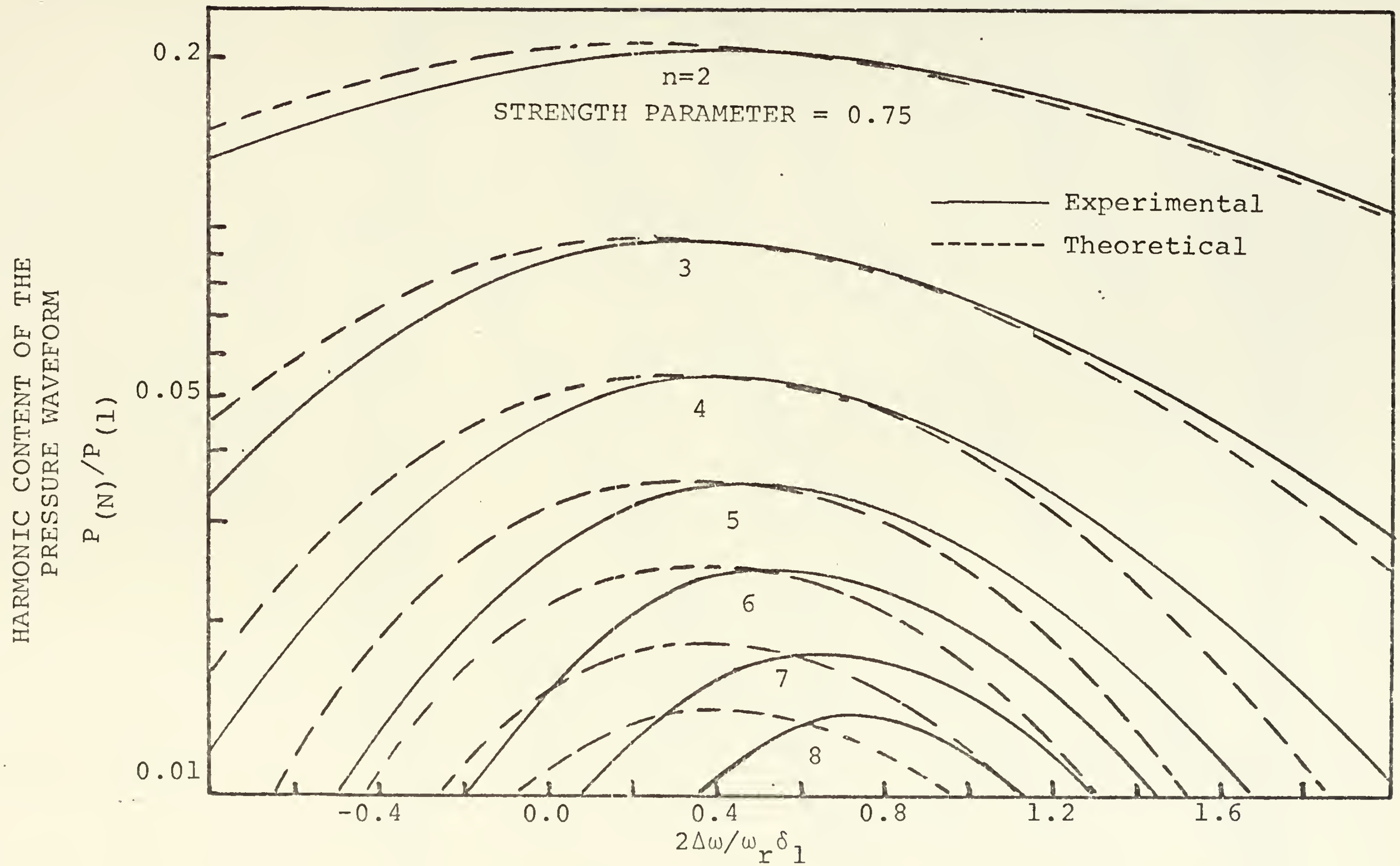
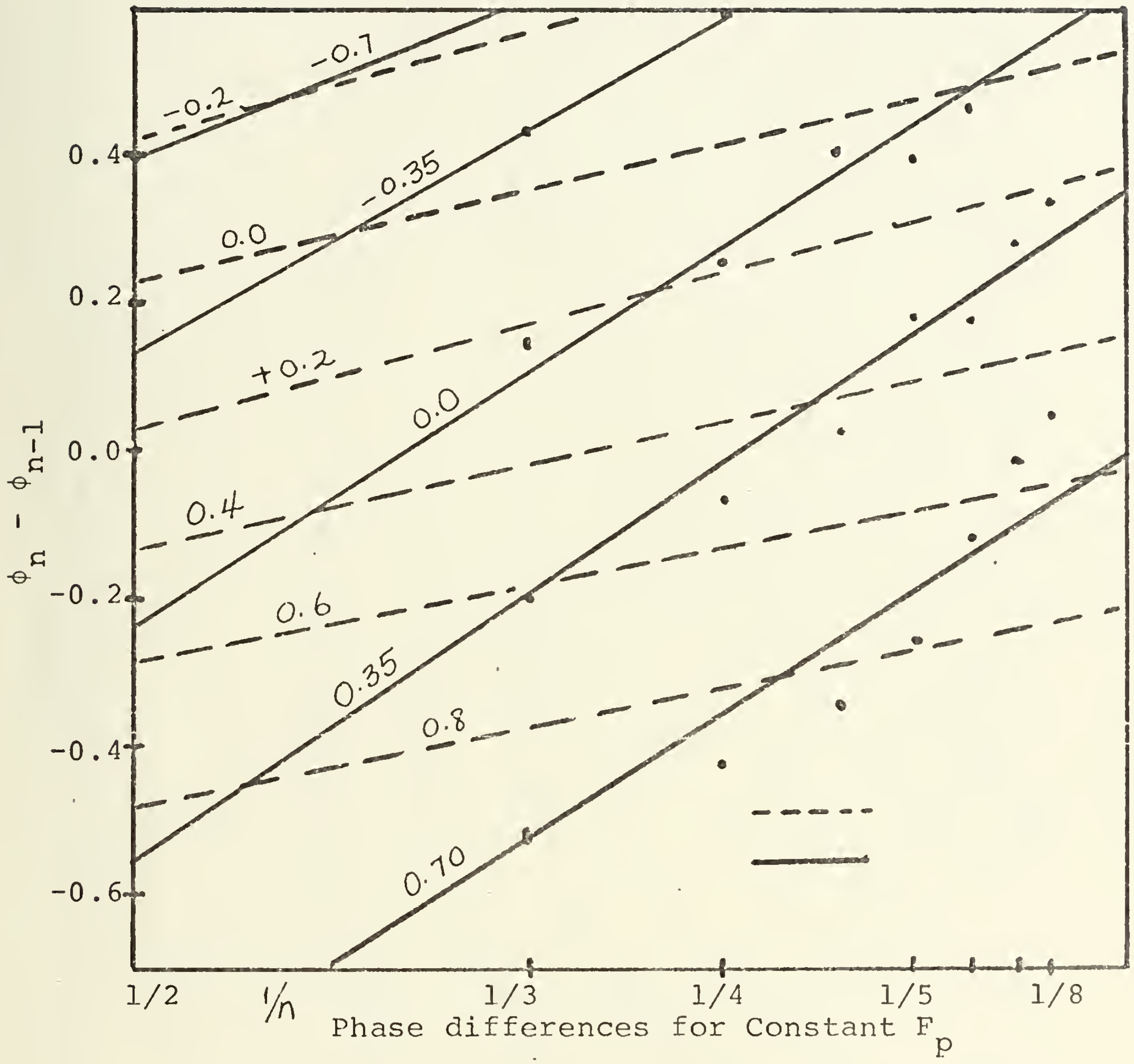


Figure 10. Harmonic Amplitudes SP = 0.75.

STRENGTH PARAMETER 0.75



Figurell. Phase Plot SP = 0.75.

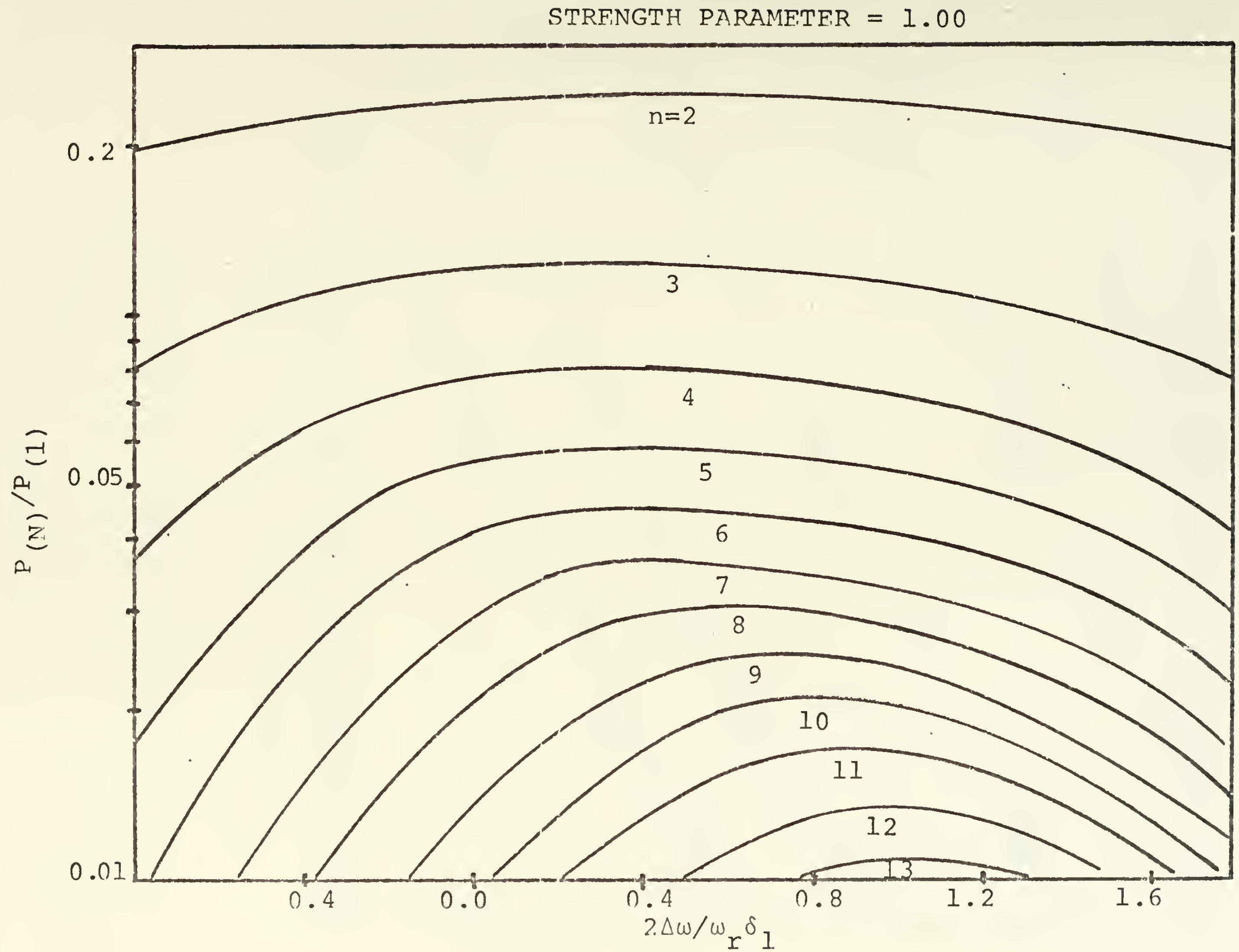


Figure 12. Harmonic Amplitudes SP = 1.00.

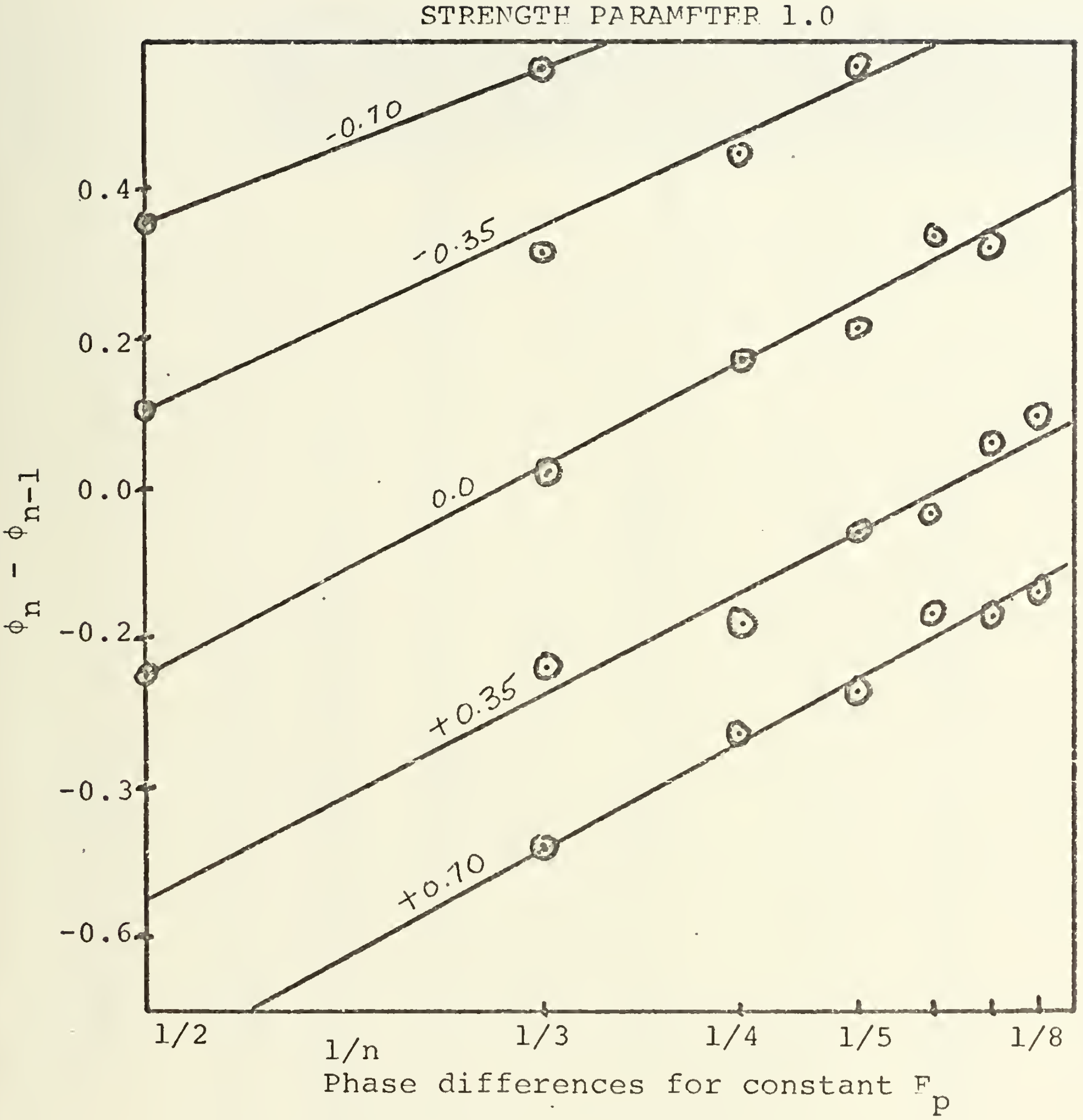


Figure 13. Phase Plot SP = 1.00

A. DIGITAL

```
//CONVERT EXEC FORTCALG,REGION.GO=130K,TIME.GO=1                        WIN00020
//FORT.SYSIN DD *                                                       WIN00030
      REAL LABEL/4H    /                                                WIN00040
      REAL*8 A(101),B(101),Y(1100),ITITLE(12),SCALE                     WIN00050
      DIMENSION INDATA(3600),DATA(1100,3),YY(1100),C(1100),             WIN00060
     1KNEW(4),X(1100)                                                   WIN00070
      READ(5,15) (ITITLE(I),I=1,12)                                     WIN00080
   15 FORMAT(6A8)                                                       WIN00090
C***********************************************************************WIN00100
C     SET THE VALUE OF STRENGTH PARAMETER.                              WIN00110
      SP=0.75                                                           WIN00120
C     SET THE NUMBER OF CHANNELS ON TAPE                                WIN00130
      NC=1                                                              WIN00140
C     SET THE NUMBER OF HARMONICS TO BE CALCULATED.                     WIN00150
      NH=10                                                             WIN00160
C     SET NDRAW TO 1 FOR CALCOMP PLOT.                                  WIN00170
      NDRAW=0                                                           WIN00180
C     SET NREC TO THE NUMBER OF RECORDS TO BE AVERAGED.                 WIN00190
C***********************************************************************WIN00200
      NREC=1                                                            WIN00210
      FACTOR=100.0/(2**31-1)                                            WIN00220
      REWIND 2                                                          WIN00230
      PI=3.141592653589793                                              WIN00240
  110 NCASE=0                                                           WIN00250
C     SET THE INITIAL RECORD COUNT AND BEGIN A NEW CASE.                WIN00260
  111 J=0                                                               WIN00270
      NCASE=NCASE+1                                                     WIN00280
      DO 112 I=1,NC                                                     WIN00290
  112 KNEW(I)=0                                                         WIN00300
C     PROCESS A NEW RECORD                                              WIN00310
C     STEP THE RECORD COUNT AND ZERO THE CHANNEL NUMBER                 WIN00320
    1 J=J+1                                                             WIN00330
      K=0                                                               WIN00340
      NSAMP=1090                                                        WIN00350
      NP=NC*NSAMP                                                       WIN00360
C     READ THE TAPE                                                     WIN00370
      READ(2,2,END=12,ERR=10)  (INDATA(I), I=1,NP)                      WIN00380
    2 FORMAT(109(10A4))                                                 WIN00390
C     READ IN THE PARTICULAR FREQUENCY AND SAMPLE COUNT FOR THIS CASE.  WIN00400
      READ(5,555) NSAMP,FREQ                                            WIN00410
  555 FORMAT(I4,F8.1)                                                   WIN00420
C     CONVERT THE SEVEN TRACK TAPE FOR USE IN THE IBM 360 SYSTEM        WIN00430
```

```
      CALL FORM(INDATA)
C     PROCESS A NEW CHANNEL
   35 CONTINUE
C     STEP THE CHANNEL COUNT
      K=K+1
C     CONVERT THE DATA TO DECIMAL
      TSCALE=0.0
      DSCALE=0.5
      DO 4 I=1,NSAMP
C     ZERO THE DATA VECTOR BEFORE ADDING THE RECORDS TOGETHER.
      IF(J.EQ.1) DATA(I,K)=0.0
      X(I)=INDATA(NC*(I-1)+K)*FACTOR
C     FIND THE NEGATIVE GOING AXIS CROSSING POINT
      IF(ABS(X(I)).GT.DSCALE) GO TO 4
      IF(I.GT.10) GO TO 405
      DHP    =INDATA(NC*(I+8)+K)*FACTOR
      IF(DHP.GT.0.0) GO TO 4
      GO TO 404
  405 DLP    =INDATA(NC*(I-9)+K)*FACTOR
      IF(DLP.LT.0.0) GO TO 4
  404 DSCALE=ABS(X(I))
C     KEEP TRACK OF NEWI TO USE IN FINDING THE PHASE DIFFERENCE
C     BETWEEN CHANNELS.
      NEWI=I
    4 IF(ABS(X(I)).GT.TSCALE) TSCALE=ABS(X(I))
      KNEW(K)=KNEW(K)+NEWI
C     REORGANIZE THE INPUT DATA SO THAT THE NEGATIVE GOING CROSSING
C     POINT IS THE FIRST DATA POINT.
      DO 44 I=1,NSAMP
      IF(I.LT.NEWI) GO TO 444
      DATA(I-NEWI+1,K)=X(I)+DATA(I-NEWI+1,K)
      GO TO 44
  444 DATA(NSAMP-NEWI+1+I,K)=X(I)+DATA(NSAMP-NEWI+1+I,K)
   44 CONTINUE
C     DETERMINE IF MORE RECORDS ARE TO BE READ BEFORE PROCESSING.
      IF(K.LT.NC) GO TO 35
      IF(J.LT.NREC) GO TO 1
C***********************************************************************
C
C     ONE CASE HAS BEEN READ IN TO THE DATA VECTOR AND IS READY TO
C     BE PROCESSED AT THIS POINT.
C
C***********************************************************************
C     ZERO THE CHANNEL COUNT
      K=0
C     SET THE CHANNEL COUNT AND THE INITIAL SCALE AND EXTRACT THE DATA
C     FOR THE CURRENT CHANNEL NUMBER.
   53 K=K+1
```

```
      SCALE=0.01
      DO 55 I=1,NSAMP
      Y(I)=DATA(I,K)
   55 IF(DABS(Y(I)).GE.SCALE) SCALE=DABS(Y(I))
C     LABEL THE CHANNEL NUMBER
      WRITE(6,45) NCASE,K
   45 FORMAT('1','CASE NUMBER',I3,6X,'CHANNEL NUMBER',I3,///)
C     FIND THE FOURIER COEFFICIENTS WITH SUBROUTINE FOX.
      CALL FOX(Y   ,NSAMP,NH,A,B,IER,SCALE)
      WRITE(6,6)
C     LABEL THE OUTPUT DATA
    6 FORMAT(1X,'HARMONIC',7X,'A(N)',10X,'B(N)',14X,'C(N)',10X,'PHI(N)',
     110X,'A(N)/A(1)')
      C(1)=A(1)
C     CHANGE THE COEFFICIENTS TO THE FORM....A(N)SIN(NWT+PHI(N)))
      DO 8 I=1,NH
      IF(I.LT.2) GO TO 7
      C(I)=DSQRT(A(I)**2+B(I)**2)
      PHI=DATAN2(A(I),B(I))
    7 CONTINUE
      IF(I.EQ.1) PHI=0.0
C     NUMBER THE HARMONICS
      N=I-1
      RATIO=0.0
      IF(I.EQ.1) GO TO 8
C     WRITE THE OUTPUT DATA
      RATIO=C(I)/C(2)
    8 WRITE(6,9)(N,A(I),B(I),C(I),PHI,RATIO)
    9 FORMAT(I4,5F16.4)
C     FIND THE PHASE DIFFERENCE BETWEEN THE PRESSURE (CH1) AND THE
C     ACCELEROMETER(CH2)
      IF(K.EQ.2)DEL= IABS(KNEW(1)-KNEW(2))*360.0/(NREC*NSAMP)
C     DETERMINE THE NUMBER OF POINTS TO BE USED IN CALL DRAW SUBROUTINE
      KK= 1+NSAMP/900
      IF(NDRAW.EQ.0) KK=1
C     DETERMINE THE POINTS TO BE PLOTTED FROM THE ORIGINAL NORMALIZED,
C     SHIFTED DATA AND CREAT AN ADDITIONAL SET OF POINTS FROM
C     THE FOURIER COEFFICIENTS.
      L=1
      SQ=0.0
      Q=0.0
      DO 95 I=1,NSAMP,KK
      DD=0.0
      DO 97 KH=1,NH
      THETA=(KH-1)*((I)*2.0*PI/FLOAT(NSAMP))
   97 DD=DD+A(KH)*COS(THETA)+B(KH)*SIN(THETA)
C     RECONSTRUCTED DATA ORDINATES, X AXIS POINTS, AND ORIGINAL
C     ORDINATES
```

```
      C(L)=DD
      X(L)=360.0*FLOAT(I-1)/FLOAT(NSAMP-1)
      YY(L)=Y(I)/ SCALE
C     CALCULATE THE AVERAGE DIFFERENCE BETWEEN TNE INPUT POINTS AND
C     THE RECONSTRUCTED POINTS.  DO THE SAME FOR THE RMS ERROR.
      DELTA=C(L)-YY(L)
      Q=DELTA+Q
      SQ=DELTA**2+SQ
C     DETERMINE THE NUMBER OF POINTS IN THE PLOTP SUBROUTINE
   95 L=L+1
      AQ=Q/FLOAT(L-1)
      RMS =SQRT(SQ/FLOAT(L-1))
      WRITE(6,950) AQ,RMS,FREQ,NSAMP
  950 FORMAT('0','AVERAGE DIFFERENCE',F10.6,//,' RMS ERROR',F7.4,//,
     2' FREQUENCY',F7.2,//,' NUMBER OF SAMPLES',I6)
C     CORRECT THE NUMBER OF PLOT POINTS
      L=L-1
C
C     PLOT ON ONE GRAPH THE ORIGINAL DATA AND THE RECONSTRUCTED DATA
C     FOR EACH CHANNEL.
C     TO SCALE THE PLOTP ROUTINE SELECT TWO POINTS AND SET THEM
C     TO THE DESIRED EXTREMUM VALUES.
      NDRAW=0
      IF(NDRAW.EQ.0) GO TO 91
      ITITLE(6)=SP
      ITITLE(9)=FREQ
      ITITLE(11)=NSAMP
      CALL DRAW(L,X,YY,1,0,LABEL,ITITLE,60.0,0.4,3,0,2,2,6,6,1,LAST)
      CALL DRAW(L,X,C,3,0,LABEL,ITITLE,60.0,0.4,3,0,2,2,6,6,1,LAST)
   91 CONTINUE
      YY(1)=1.2
      YY(1+120*KK)=-1.2
      CALL PLOTP(X,YY,L,1)
      CALL PLOTP(X,C,L,3)
C
C
      IF(K.NE.2) GO TO 99
      WRITE(6,98) SP,DEL
   98 FORMAT('0','STRENGTH PARAMETER=',F6.3,'PHASE=',F7.2)
C     CHECK THE CHANNEL NUMBER AND DETERMINE IF ANOTHER CHANNEL IS
C     TO BE PROCESSED OR A NEW CASE IS TO BE READ.
   99 CONTINUE
      IF(K.EQ.NC) GO TO 111
      GO TO 53
   10 WRITE(6,11) J
   11 FORMAT ('0',5X,'READ ERROR, RECORD NO.',I3)
      GO TO  1
   12 WRITE(6,13) J
```

```
 13 FORMAT ('0',5X,'END OF TAPE, RECORD NO.=',I3)
 14 CONTINUE
    STOP
    END

    SUBROUTINE FOX(FNT , NSAMP,M,A,B,IER,SCALE)
    IMPLICIT REAL*8(A-H),REAL*8(O-Z)
    DIMENSION A(1),B(1),FNT(1)
    IER=0
    N=(NSAMP-1)/2
 20 IF(M) 30,40,40
 30 IER=2
    RETURN
 40 IF(M-N) 60,60,50
 50 IER=1
    RETURN
 60 AN=N
    MP1 = M+1
    DZ = 0.D0
    D1 = 1.D0
    D2 = 2.D0
    COEF = D2/(D2 * AN + D1)
    CONST = 3.141592653589793 * COEF
    S1 =  DSIN(CONST)
    C1 =  DCOS(CONST)
    C = D1
    S = DZ
    J=1
    FNTZ=FNT(1)/SCALE
 70 U2 = DZ
    U1 = DZ
    I=2*N+1
 75 U0=FNT(I)/SCALE+D2*C*U1-U2
    U2=U1
    U1=U0
    I=I-1
    IF(I-1) 80,80,75
 80 A(J)=COEF*(FNTZ+C*U1-U2)
    B(J)=COEF*S*U1
    IF(J-(M+1)) 90,100,100
 90 Q=C1*C-S1*S
    S=C1*S+S1*C
    C=Q
    J=J+1
    GO TO 70
100 A(1) = A(1) / D2
    RETURN
```

```
      END                                                                WINO2340
//ASM.SYSIN   DD  *                                                      WINO2360
FORM     START 0                                                         WINO2370
         STM    14,12,12(13)                                             WINO2380
         BALR   6,0                                                      WINO2390
         USING *,6                                                       WINO2400
         USING DATA,7                                                    WINO2410
         SR     7,7                                                      WINO2420
         L      11,=F'1090'                                              WINO2430
         L      12,0(1)                                                  WINO2440
LOOP     L      2,NUM(12)                                                WINO2450
         LR     3,7                                                      WINO2460
         SRDL   2,6                                                      WINO2470
         SRL    2,2                                                      WINO2480
         SRDL   2,6                                                      WINO2490
         SRL    2,2                                                      WINO2500
         SRDL   2,6                                                      WINO2510
         SRL    2,2                                                      WINO2520
         SRDL   2,6                                                      WINO2530
         ST     3,NUM(12)                                                WINO2540
         LA     12,4(12)                                                 WINO2550
         BCT    11,LOOP                                                  WINO2560
         LM     2,12,28(13)                                              WINO2570
         MVI    12(13),X'FF'                                             WINO2580
         BCR    15,14                                                    WINO2590
DATA     DSECT                                                           WINO2600
NUM      DS     1F                                                       WINO2610
         END                                                             WINO2620
//GO.FT02F001 DD UNIT=OCO,VOL=SER=WINN,LABEL=(6,NL),                     WINO2640
//       DISP=(OLD,KEEP),DCB=(DEN=1,RECFM=F,BLKSIZE=4360)                WINO2650
//GO.SYSIN   DD *                                                        WINO2660
J.R. WINN 1542    STRENGTH PARAMETER=                                    WINO2670
FREQUENCY=                                    SAMPLES                    WINO2680
1063 94.1                                                                WINO2690
1063 94.1                                                                WINO2700
1063 94.1                                                                WINO2710
1055 94.8                                                                WINO2720
1055 94.8                                                                WINO2730
1055 94.8                                                                WINO2740
1065 93.9                                                                WINO2750
1065 93.9                                                                WINO2760
1065 93.9                                                                WINO2770
1051 95.2                                                                WINO2780
1051 95.2                                                                WINO2790
1051 95.2                                                                WINO2800
1059 94.5                                                                WINO2810
1059 94.5                                                                WINO2820
1059 94.5                                                                WINO2830
```

B. ANALOG CONVERSION

```
      DIMENSION IBUF(2048,2), LOCB(-1'1), MAXBS(-1'1)
      INTEGER RECNUM
      NAMELIST NREC, NSAMP, NCHAN, ITAPE,NDEL
1     INPUT(101)
      LOCB(-1)=LOCF(IBUF(1,1))
      LOCB(1)=LOCF(IBUF(1,2))
      NWORDS=NSAMP*NCHAN
      MAXBS(-1)=LOCB(-1)+NWORDS-1
      MAXBS(1)=LOCB( 1)+NWORDS-1
      IF(SENSE SWITCH 6)2,15
2     NB=1
      IND=0
      RECNUM=0
      NEWBUF=LOCB(1)
      MAXB=MAXBS(-1)
      CALL ADSTART(NCHAN,LOCB(-1),NEWBUF,MAXB,RECNUM,11S)
      MAXB=MAXBS(1)
3     IF(TEST(1).GT.0)GO TO 3
      CALL ENABLE
5     CONTINUE
10    GO TO 5
11    IF(IND.EQ.1)GO TO 90
      IF(TEST(1).GT.0.OR.RECNUM.GE.NREC)CALL DISABLE
      NB=-NB
      NEWBUF=LOCB(NB)
      MAXB=MAXBS(NB)
      IND=1
      CALL BUFFEROUT(ITAPE,1,IBUF(1,(3+NB)/2),NWORDS,IND)
      IF(TEST(1).LT.0.AND.RECNUM.LT.NREC)GO TO 5
      CALL ADSTOP
X     CALL PROCESS(IBUF,NSAMP,NCHAN,2S)
      OUTPUT(101)RECNUM
15    OUTPUT(101)-OPTION=(I1)-
      READ(101,100)NOPT
100   FORMAT(I1)
      GO TO(1,2,30,40,50,60,70)NOPT
30    ENDFILE(ITAPE)
      OUTPUT(101)-EOF-
      GO TO 15
40    REWIND(ITAPE)
      GO TO 15
50    OUTPUT(101)-SKIPFILES=(I4)-
      READ(101,101)NF
101   FORMAT(I4)
      DO 55 I=1,NF
51    CALL BUFFERIN(ITAPE,1,IBUF(1,1),1,IND)
52    IF(IND.LT.2)GO TO 52
      IF(IND.NE.3)GO TO 51
```

```
55    CONTINUE
      OUTPUT(101)NF
      GO TO 15
60    OUTPUT(101)-NUMWORDS TO LIST=(I4)-
      READ(101,101)NW
      WRITE(101,105)NW,NCHAN
105   FORMAT(- WRITE - I4 - WORDS, - I2 - AT A TIME-)
      IND=1
      CALL BUFFERIN(ITAPE,1,IBUF(1,1),NWORDS,IND)
66    IF(IND.EQ.1)GO TO 66
62    GO TO(62,63,64,65)IND
63     WRITE(6,102)
102   FORMAT(1H1)
      DO 631 I=1,NW,NCHAN
      WRITE(6,104)(IBUF(J,1),J=I,I+NCHAN-1)
104    FORMAT(12010)
631   CONTINUE
      GO TO 15
64     OUTPUT(101)-EOF READ-
      GO TO 15
65     OUTPUT(101)-READ ERR-
      GO TO 63
70    OUTPUT(101)-START ANALOG RECORDER-
      OUTPUT(101)-TYPE * C/R TO CONTINUE-
      INPUT(101)
      IND=1
      CALL BUFFERIN(ITAPE,1,IBUF,NWORDS,IND)
76    IF(IND.EQ.1)GO TO 76
71    GO TO(71,72,64,74)IND
72    DO 73 I=1,NWORDS
73    IBUF(I,1)=IBUF(I,1)/2**10
      DO 75 I=1,NWORDS,NCHAN
      DO 75 J=1,NCHAN
      CALL DAC(J,IBUF(I+J-1,1))
      N=NDEL
      CALL DELAY
75    CONTINUE
      GO TO 15
74    OUTPUT(101)-READ ERROR-
      GO TO 72
90    CALL DISABLE
      CALL ADSTOP
      OUTPUT(101)-RATE ERR-,RECNUM
      GO TO 15
      END
```

```
      SUBROUTINE PROCESS(IB,NS,NC,IS)
      DIMENSION IB(NC,NS),MEAN(10),SIGMA(10)
      REAL MEAN
      SCALE=100./2**23
      DO 1 I=1,10
1     SIGMA(I)=MEAN(I)=0.
      DO 100 I=1,NC
      DO 10 J=1,NS
10    MEAN(I)=MEAN(I)+SCALE*IB(I,J)
      MEAN(I)=MEAN(I)/NS
      DO 20 J=1,NS
20    SIGMA(I)=SIGMA(I)+(MEAN(I)-IB(I,J)*SCALE)**2
      SIGMA(I)=SQRT(SIGMA(I)/NS)
      WRITE(6,1000)I,MEAN(I),SIGMA(I)
1000  FORMAT($ CHAN $I2$  MEAN=$F8.4$  SIGMA=$F8.4/)
100   CONTINUE
30    IF(TEST(1).LT.0)GO TO 30
      IF(TEST(2).GT.0)RETURN
      RETURN IS
      END
¬META9300 SI,LO,GO
$ADSTART PZE
        BRM     9SETUPN
        PZE     6
NCH     PZE
BUF     PZE
NEWBUF  PZE
MAXB    PZE
RECNUM  PZE
NEXLOC  PZE
        LDA     ENDBRM
        XMA     040
        STA     SV040
        LDA     INTBRM
        XMA     052
        STA     SV052
        LDA     *NCH
        STA     INCP
        ADD     =COMM
        COPY    (5,1)
        STA     COMLOC
        LDA     *NCH
        LLSA    15
        ADD     =COMM
        STA     0,1
        STA     CONTR
        LDA     *BUF
        STA     COMM
```

```
        LDA      *MAXB
        STA      MAX
         SKR      NFULL
         BRU      $-1
         EOM      034001
         POT      CONTR
         BRR      ADSTART
INTBRM   BRM      ADFAST
ENDBRM   BRM      ENDAD
SVC40    PZE
ENDAD    PZE
         DIR
         HLT
         EOM      034001
         POT      CONTR
         EIR
         BRC      *ENDAD
$ADSTOP  PZE
         LDA      SVC40
         STA      040
         LDA      SV052
         STA      052
         STZ      *COMLOC
         MPO      ADSTOP
         BRR      ADSTOP
COMLOC   PZE
$ADFAST  PZE
         DIR
         STD      SVAB
         SKN      NFULL
         BRU      NXTBUF
         LDP      INCR
         ADD      COMM
         STA      COMM
         ADD      INCR
        SKL      MAX
         STB      NFULL
         LDP      SVAB
         EIR
         BRC      *ADFAST
NXTBUF   LDA      *NEWBUF
         STA      COMM
         LDA      *MAXB
         STA      MAX
         SKR      NFULL
         BRU      $-1
         MPO      *RECNUM
         LDP      SVAB
```

```
         EIR
         BRC       *NEXLOC
COMM     RES       33
CONTR    PZE
MAX      PZE
SVAB     RES       2
INCR.    DATA      6,0
NFULL    PZE
SV052    PZE
         END
```

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13. ABSTRACT

Finite-amplitude standing waves in air at ambient conditions contained in a rigid-walled cylindrical tube with a large length-to-diameter ratio were experimentally investigated. The pressure waveform at the end of the tube was digitized and Fourier analyzed on an IBM 360 digital computer. Amplitudes and phases were obtained for all harmonics with amplitudes greater than 1% of the fundamental for strength parameters from 0.25 to 1.00 and for frequency parameters from -0.8 to 2.0. The strength and frequency parameters are defined as MbQ and $2\Delta f/\Delta f$ respectively, where M is the Mach number of fundamental, b the nonliniarity parameter, Q the quality factor of the resonator, Δf the frequency away from fundamental resonance, and Δf the band width at the half-power points. When these results are compared to the theoretical model of Coppens and Sanders, it is seen that while the theory accurately predicts the magnitude and shape of the harmonic content, it consistently underestimates the frequency at which each harmonic peaks. In addition, the theory fails to predict, except qualitatively, the phase angles.

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